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PANEL ESTIMATES OF A TWO-TIERED EARNINGS FRONTIER

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SUMMARY

This paper uses panel data to estimate a two-tiered instead of a one-tiered frontier model. The innovation is to develop a two-step maximum likelihood procedure yielding consistent estimates of inefficiency, while at the same time accounting for heterogeneity. The model is applied by estimating a 'two-tiered' earnings function to obtain indices of worker and firm incomplete labour market wage information using panel data from the Panel Study of Income Dynamics (1969–84). The estimation preserves the traditional quadratic age–earnings profile, but measures the extent to which employers often pay more than necessary to hire a worker (incomplete employer information), while at the same time, employees often accept wages less than they could otherwise command (incomplete employee information). The results indicate that employees acquire less information than employers.

1. INTRODUCTION

Frontier estimation entails measuring the degree to which data lie below a boundary function. Early work involved mathematical programming (Farrell, 1957) in which technical efficiency could be measured by comparing observed with potential performance. Statistical analyses followed (Aigner *et al.*, 1977; Meeusen and van den Broeck, 1977) incorporating composed error terms thereby enabling performance to be viewed as stochastic. In these latter models a negative one-sided error augmented the normally distributed traditional disturbance term. These techniques were applied mostly in production function models to measure how much output often lags potential levels.

In Polachek and Yoon (1987) we extended this statistical framework by developing an empirically tractable 'two-tiered' model. Two one-sided errors, one positive and one negative, were introduced to augment the normally distributed two-sided error term. We applied this model to earnings functions because when measuring earnings it is important to know both the extent employers often pay more than necessary to get a worker, as well as the extent to which employees at the same time will accept a wage less than they otherwise could command. We interpreted the two one-sided errors to respectively represent employer and employee 'ignorance' of each other's reservation wages, and hence to measure employer and employee incomplete information.[1] This interpretation is consistent with search theory's predictions, but it is not critical to the work presented herein.

[1] Also see Gaynor and Polachek (1994) which extends this model to the health care market, and gives a more precise description of the search theory implications.

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One problem with both these two-tiered frontier as well as the traditional one-sided frontier estimation models is that the inefficiency term may include unobserved heterogeneity so that it is not a pure inefficiency measure. Cross-sectional data contaminated by unobserved heterogeneity make it impossible for the researcher to identify inefficiency. In the case of a single one-sided error model of firm inefficiency, the production performance of a particular firm based only on *observed* characteristics may exceed output produced by more efficient firms located elsewhere because unobserved but invariant location-specific characteristics such as poor access to transportation or unfavourable climate may dominate the efficiency edge of the latter firms. Similarly, in our two-tiered frontier case with two one-sided inefficiency errors, unobserved worker heterogeneity also can bias upward the employer and employee ignorance measures.

Our previous paper handled this problem by looking at *relative* ignorance measures across demographic strata. For example, we found that workers receiving unemployment insurance (UI) exhibited more complete job market information (were more efficient in obtaining a higher wage) than workers who had not received UI. Since there is *little* reason to expect unmeasured heterogeneity to vary between these strata (e.g., whether a worker received UI), we interpreted differences in our information measures between strata to reflect *relative* rather than *absolute* measures of incomplete information across labour markets. However, another remedy to adjust for unmeasured heterogeneity is to use panel data. This is the approach we pursue here.

Panel data follow the same individuals over time. With panel data, intercepts for each individual can be identified.[2] Pitt and Lee (1981) as well as Schmidt and Sickles (1984) interpret these intercept differences as measuring deviations in firm-specific *efficiency*. Cornwell, *et al.* (1990) extend this model to consider time-varying intercepts, and hence dynamic changes in efficiency.

Unobserved heterogeneity literature, on the other hand, offers a different interpretation. Proponents of this literature argue that intercept differences measure unobserved person specific heterogeneity. Distinguishing between unobserved heterogeneity and inefficiency is important because, as indicated earlier, factors other than inefficiency can cause some firms' output to be lower. Similarly in our two-tier example these factors as well as unobserved worker ability can affect wages independent of the effects of a worker's or a firm's efficiency in gathering labour market information.

This paper makes an inroad into the problem of distinguishing between individuals' unobserved heterogeneity and inefficiency by devising a maximum likelihood (ML) method to make such a distinction at least at the market level. We use a computationally feasible two-step approach. First, individual-specific constants are computed by conventional procedures. Second, results from this first step are embedded into the maximum likelihood estimation. This approach is in the spirit of Kumbhakar and Hjalmarsson (forthcoming) who also use a multi-stage fixed-effects model.[3] However, in this paper we extend the panel data fixed-effects procedure to the more general two-tier frontier which incorporates the traditional one-tier frontier as a special case. In addition, our ML estimates are consistent. (See Polachek and Yoon, 1994.)

2. THE MODEL

Our past paper (1987) specifies a two-tiered earnings frontier derived from a supply and demand model. Since the derivation is readily available and not really germane to this paper's contribution, we merely provide some intuition for the final reduced-form wage equation.

[2] In fact, with a sufficiently long panel, individual differences in other parameters are also identifiable. See Polachek and Kim (1994).

[3] Another approach entails random coefficients models. See Kumbhakar (1991) and Heshmati and Kumbhakar (1994).

We define potential wage to be the equilibrium wage when both workers and firms have full information. This is determined by full information supply and demand considerations, and results in a reduced form comparable to the human capital specification. Generally, an individual i with earnings potential y^* works in firm f at earnings observed in time period t to be y^*_{ift}.[4] Potential earnings levels are determined by worker and firm characteristics x_{ift} as well as a random two-sided normally distributed (in the logs) error term u_{ift}, so that

$$y^*_{ift} = \beta' x_{ift} + u_{ift} \text{ [5]} \tag{1}$$

Observed earnings can be somewhat above or below potential earnings depending upon worker and firm information of each other's reservation wages. Incomplete worker information implies that the worker receives a wage less than the maximum of offer wages across firms, while incomplete firm information implies that the firm pays a wage higher than the minimum of reservation wages across workers. As such, observed earnings y_{ift} is expressed as follows:

$$y_{ift} = y^*_{ift} + v_{it} + w_{ft} \tag{2}$$

where $v_{it} \in (-\infty, 0)$ represents the difference between the maximum firm offer wage and the wage worker i actually receives, while $w_{ft} \in (0, \infty)$ represents the difference between the wage firm f actually pays and the minimum worker reservation wage. Substituting equation (1) into equation (2) yields

$$y_{ift} = \beta' x_{ift} + \varepsilon_{ift} \tag{3}$$

where

$$\varepsilon_{ift} = u_{ift} + v_{it} + w_{ft} \tag{4}$$

In this reduced-form equation observed earnings are related to usual 'human capital' and other 'productivity' considerations of the worker and firm (x_{ift}), as well as a three-component error term (ε_{ift}). As indicated, component one $u_{ift} \in (-\infty, \infty)$ is a typical random error, while the other two components have search theory interpretations. Component two $v_{it} \in (-\infty, 0)$ represents the difference between the maximum firm offer wage and the wage the worker actually receives. It is interpreted as incomplete employee information because it measures how much less an employee receives compared to how well the employee could do. Component three $w_{ft} \in (0, \infty)$ depicts incomplete employer information. It represents the difference between what the firm actually pays and the minimum worker reservation wage, thereby illustrating the degree to which a firm's wage exceeds the minimum necessary wage just needed to hire the employee.

With cross-sectional data, variations in the intercept term across individuals are not discernible. Identifying individual fixed effects necessitates using panel data. With panel data the error can be decomposed into individual, firm, and pure noise components. Accordingly, denote

$$v_{it} = v_i + v^*_{it}, \qquad v_i \leqslant 0 \text{ and } v^*_{it} \leqslant 0 \tag{5}$$

where v_i represents time-invariant worker-specific labour market inefficiency (ignorance) and v^*_{it} the stationary stochastic inefficiency (ignorance) whose probability distribution is common to all workers in the market. Likewise, denote

$$w_{ft} = w_f + w^*_{ft}, \qquad w_f \geqslant 0 \text{ and } w^*_{ft} \geqslant 0 \tag{6}$$

[4] For generality we define earnings to have individual, firm and time components. However, later at the estimation stage because of data limitations regarding identifying the firms at which individuals work, we are forced to drop the firm-specific component.

[5] One can treat β as time dependent to get a random coefficient framework much like *Cornwell, et al.* (1990). For simplicity and because the coefficients of earnings functions are usually assumed constant, we omit this generalization.

where similarly w_f is time-invariant firm-specific labour market inefficiency (ignorance) and w^*_{ft} is the stationary stochastic labour market inefficiency (ignorance) whose probability distribution is common to all firms in the market. As implied by the notation, we assume v_i and w_f are fixed constants for worker i and firm f, and v^*_{it}, w^*_{ft} and u_{ift} are independently and identically distributed across workers (i), firms (f), and time (t).[6]

Substituting equations (4)–(6) into equation (3) yields a fixed-effect 'two-tiered' earnings frontier:

$$\begin{aligned} y_{ift} &= \beta_0 + \beta' x_{ift} + u_{ift} + v_{it} + w_{ft} \\ &= (\beta_0 + v_i + w_f) + \beta' x_{ift} + u_{ift} + v^*_{it} + w^*_{ft} \\ &= \alpha_{if} + \beta' x_{ift} + u_{ift} + v^*_{it} + w^*_{ft} \\ &= \alpha_{if} + \beta' x_{ift} + \varepsilon^*_{ift} \end{aligned} \tag{7}$$

where $\alpha_{if} = \beta_0 + v_i + w_f$; $\varepsilon^*_{ift} = u_{ift} + v^*_{it} + w^*_{ft}$; and x_{ift} is now redefined to denote the list of regressors excluding the unity. Among the stochastic terms, $v^*_{it} \leq 0$, $w^*_{ft} \geq 0$, and $u_{ift} \in (-\infty, +\infty)$ are purely random noise. The intercept term α_{if} contains the individual worker and firm permanent inefficiencies v_i and w_f as well as the effects of some of the regressors which are unobservable and/or omitted from the list x_{ift}. Thus model (7) is a fixed-effect one containing the nuisance parameters (α_{if}: $i = 1, \ldots, N$; $f = 1, \ldots, F$), where N denotes the number of workers and F the number of firms. The model also incorporates the marketwide worker and firm (labour supply and demand) inefficiency given by the dispersion of v^*_{it} and w^*_{ft}. The identification of the individual level inefficiency (v_i and w_f) may not be feasible due to the presence of unobserved and omitted variables. However, equation (7) allows one to identify the inefficiencies at the market level, as will be shown below.

Frontier estimation usually assumes a normally distributed two-sided error and either half-normal or exponential one-sided errors. (See Aigner *et al.*, 1977; Gong and Sickles, 1989.) For analytical tractability, we assume exponentially distributed one-sided errors along with a normally distributed error. As such, $u_{ift} \sim N(0, \sigma^2_u)$, $-v^*_{it}$ has an exponential distribution with mean μ_v, and w^*_{ft} an exponential distribution with mean μ_w. Obviously, the variance of v^*_{it} and w^*_{ft} are obtained by squaring their means, μ^2_v and μ^2_w.

Suppressing the subscripts, the marginal density of ε^*_{ift} is given by (see Polachek and Yoon, 1985, or Yoon and Polachek, 1987 for a proof)

$$\begin{aligned} g(\varepsilon^*) = \frac{1}{\mu_v + \mu_w} \cdot \exp\left(\frac{\varepsilon^*}{\mu_v} + \frac{\sigma^2_u}{2\mu^2_v}\right) \\ \times \left\{1 - \Phi\left(\frac{\varepsilon^*}{\sigma_u} + \frac{\sigma_u}{\mu_v}\right) + \left(1 - \Phi\left(\frac{-\varepsilon^*}{\sigma_u} + \frac{\sigma_u}{\mu_w}\right)\right)\right. \\ \left. \times \exp\left[-\frac{1}{2}\left(\frac{2\varepsilon^*}{\sigma_u} + \sigma_u\left(\frac{1}{\mu_v} - \frac{1}{\mu_w}\right)\right) \cdot \sigma_u\left(\frac{1}{\mu_v} + \frac{1}{\mu_w}\right)\right]\right\} \end{aligned} \tag{8}$$

[6] One can introduce additional structure about the nature of v^*_{it} and w^*_{ft}. For example, one can model these stochastic inefficiency (incomplete information) terms to contain a transitory component. However, how incomplete information varies with time is not obvious. Incomplete information could decrease with experience as individuals continue to gather information over the life cycle, yet increase as information depreciates (Polachek and Horvath, 1977). Our point is to present one fundamental specification, and leave for future work experimentation with alternatives.

where Φ denotes the cumulative density function of the standard normal random variate.

Given panel data $\{y_{ift}, x_{ift}\}$, the likelihood function is:

$$L(y; \alpha, \beta, \sigma_u, \mu_v, \mu_w) = \prod_{i=1}^{N} \prod_{f=f_i}^{F_i} \prod_{t=t_{if}^b}^{T_{if}^e} g(\varepsilon_{ift}^*)$$

$$= \prod_{i=1}^{N} \prod_{f=f_i}^{F_i} \prod_{t=t_{if}^b}^{T_{if}^e} g(y_{ift} - \alpha_{if} - \beta' x_{ift}) \tag{9}$$

where f_i and F_i denote the first and last firms of worker i during the panel; t_{if}^b and T_{if}^e are the beginning and ending periods for the job match between worker i and firm f; $\alpha = (\alpha_{if}: i = 1, \ldots, N; f = 1, \ldots, F)'$; α_{if} is the worker–firm-specific intercept; and the density g is given in equation (8) above.

3. ESTIMATION

The Panel Study of Income Dynamics (PSID) is by far the largest and longest-running national panel data. Probably it is the one most used in longitudinal labour market studies. For this reason, we adopt the PSID data here. We use annual data from 1968 to 1984. Year effects are incorporated by deflating wages by the CPI index.

Our sample data consist of 838 workers who worked an identical number of periods T. Workers can and do move across firms, but in the PSID one cannot identify specific firms. Similarly, given the data, it is unlikely that any two individuals were employed at the same firm.[7] This limitation precludes distinguishing between firm-specific and individual-specific effects, as both *together* are subsumed in the intercept. It implies that the likelihood function (9) can be rewritten as

$$L(y; \alpha, \beta, \sigma_u, \mu_v, \mu_w) = \prod_{i=1}^{N} \prod_{t=1}^{T} g(\varepsilon_{it}^*)$$

$$= \prod_{i=1}^{N} \prod_{t=1}^{T} g(y_{it} - \alpha_i - \beta' x_{it}) \tag{10}$$

where the f subscript is dropped since the specific firm is now indistinguishable from the worker, and where α is a $N \times 1$ vector of intercepts that incorporate *both* worker and firm heterogeneity. It could be thought of as a worker's overall intercept that includes specific effects from *each* firm where the worker has been employed. Full maximization of likelihood function (10) provides consistent estimators of all the parameters. A proof is given in Polachek and Yoon (1994) by showing that the expected value of the derivative of the likelihood function with respect to each parameter is zero.

Maximizing function (10) directly is impractical because α contains N elements. For tractability, we adopt a two-step approach. Step 1 entails consistent least-squares estimation of β based on the mean deviation model. Step 2 maximizes the likelihood function using the β

[7] We are unaware of any other panel data identifying *both* employers and employees, let alone one in which employers had employees in common.

estimate of Step 1 to replace α with its consistent estimate.[8] Specifically for worker i, who is employed over the period $t = 1, \ldots, T$, the earnings equation (7) can be rewritten as

$$y_{it} = a_i + \beta' x_{it} + e_{it} \tag{11}$$

where

$$a_i = \alpha_i - \mu_v + \mu_w \tag{12}$$

and

$$e_{it} = u_{it} + (v^*_{it} + \mu_v) + (w^*_{it} - \mu_w) \tag{13}$$

The unit of observation is (i, t), since now the data are *worker* rather than worker–firm based, which are unavailable. This implies that firm f is indistinguishable from worker i. To reflect this, w^*_{ft} is rewritten as w^*_{it} so that v^*_{it} and w^*_{it} represent worker and firm stationary stochastic incomplete information for each worker i. As such, w^*_{it} is the tth time-period employer's incomplete information with regard to worker i, v^*_{it} is worker i's incomplete information with respect to the tth time-period employer, and the error term e_{it} has a zero mean. As indicated above, the coefficient a_i is the worker-specific intercept. It represents each worker's fixed-effect and incorporates the time-invariant specific effects from each firm where the worker has been employed.

Noting that β is common for all workers, we apply least-squares to the mean-deviation model to get a consistent estimate of β:

$$y'_{it} = \beta' x'_{it} + e'_{it} \qquad \text{for } i = 1, \ldots, N \text{ and } t = 1, \ldots, T \tag{14}$$

where the primes on the variables denote mean deviations (not vector transposes) so that $y'_{it} = y_{it} - \bar{y}_i$, $x'_{it} = x_{it} - \bar{x}_i$, $e'_{it} = e_{it} - \bar{e}_i$; $\bar{y}_i = \Sigma y_{it}/T$, $\bar{x}_i = \Sigma x_{it}/T$, and $\bar{e}_i = \Sigma e_{it}/T$. The worker-specific intercept is computed from the least squares estimate of β, $\hat{\beta}$, as follows:

$$\hat{a}_i = \bar{y}_i - \hat{\beta} \cdot \bar{x}_i \tag{15}$$

Since $\alpha_i = a_i + \mu_v - \mu_w$, the likelihood function (10) conditional on α being $\dot{\alpha} \equiv \{\alpha_i = \hat{a}_i + \mu_v - \mu_w : i = 1, \ldots, N\}$ is expressed as[9]

$$L(y; \beta, \sigma_u, \mu_v, \mu_w \mid \dot{\alpha}) = \prod_{i=1}^{N} \prod_{t=1}^{T} g(y_{it} - \hat{a}_i + \mu_v - \mu_w - \beta' x_{it}) \tag{16}$$

Using the reparameterization

$$\Theta_u = 1/\sigma_u$$
$$\Theta_v = \sigma_u/\mu_v$$

and

$$\Theta_w = \sigma_u/\mu_w$$

[8] The success of this two-step approach hinges on $T = 17$ (the number of periods of data available to us) being sufficiently large so that we obtain consistent estimates.

[9] Correlation between $\hat{a}_i$ and $\hat{\beta}$ can bias the estimate of the variance–covariance matrix $V(\hat{\beta})$ obtained from the conditional likelihood function method. However, the magnitude of the correlation is expected to be small, because $\hat{a}_i$ uses only one individual (over T periods) while $\hat{\beta}$ uses all other $(N - 1)$ individuals in addition to the ith individual used for $\hat{a}_i$ estimation. This problem can be eliminated in the iterative approach outlined below (footnote 10), in which the likelihood function is concentrated in β alone.

we can write the log-likelihood function as

$$\log \mathrm{L} = NT \log[(\Theta_u \Theta_v \Theta_w/(\Theta_v + \Theta_w)] + \left[\Theta_u \Theta_v \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{\varepsilon}_{it}^* + (NT/2)\Theta_v^2\right]$$

$$+ \sum_{i=1}^{N} \sum_{t=1}^{T} \log\{1 - \Phi(\Theta_u \hat{\varepsilon}_{it}^* + \Theta_v) + [1 - \Phi(-\Theta_u \hat{\varepsilon}_{it}^* + \Theta_w)]$$

$$\times \exp[-\tfrac{1}{2}(2\Theta_u \hat{\varepsilon}_{it}^* + \Theta_v - \Theta_w)\cdot(\Theta_v + \Theta_w)]\} \qquad (17)$$

where

$$\hat{\varepsilon}_{it}^* = y_{it} - \hat{\alpha}_i - \beta' x_{it} = y_{it} - \hat{a}_i + \mu_v - \mu_w - \beta' x_{it}$$

Maximization of the conditional likelihood function (17) provides a consistent estimator $\hat{\Theta} = (\hat{\Theta}_u, \hat{\Theta}_v, \hat{\Theta}_w, \hat{\beta}')'$ because it is conditioned on a consistent estimator of α.[10]

4. RESULTS

The estimates are given in Table I. Column 1 contains OLS results which are used as starting values for the MLE computations. Column 2 contains the maximum likelihood results estimated without fixed effects. Finally column 3 gives fixed-effect estimates using our maximum likelihood procedure.[11] In each case the coefficients are plausible.

The wage–experience and wage–tenure gradients are quadratic as is typical. However, once account is taken of individual heterogeneity (column 3), the tenure coefficients drop dramatically (from 0·00073 to 0·00019 for the linear coefficient and −0·133E−5 to −0·543E−6 for the quadratic term). This finding is comparable to those of Altonji and Shakotko (1987) and Ruhm (1990) who claim cross-sectional studies overestimate tenure effects.[12] Because high-ability/high-wage individuals are more likely to survive in a job, cross-sections contain both a preponderance of low-ability/low-wage/low-tenure employees as well as a preponderance of high-wage/high-ability/high-tenure workers. This implies a steeper wage–tenure gradient when unobserved ability is unaccounted for as in cross-sectional analysis.

The rate of return to education is comparable to other studies. However, because panel data mitigate educational vintage effects, somewhat smaller rates of return are obtained than in Polachek and Yoon (1987) which is cross-sectional. Because education is time-invariant, no coefficients for this variable or a constant intercept can be presented in the fixed-effect model.

The most dramatic differences are in the estimated dispersion measures of the three error terms. The two-sided error dispersion (σ_u) declines the most from 0·253 to 0·032, likewise worker and firm information inefficiencies (μ_v and μ_w) decrease somewhat (0·255 to 0·194 and 0·250 to 0·178 respectively), and the log likelihood increases sharply. These findings are not

[10] To obtain an efficient estimate of Θ, one can use the following iterative approach. First, obtain the β estimate by maximizing the conditional likelihood function (17). Then rewrite $\hat{a}_i$ in terms of this conditional ML estimate $\hat{\beta}$ using the expression $a_i = \bar{y}_i - \hat{\beta}' \cdot \bar{x}_i$. Re-estimate β, maximizing this concentrated likelihood function, and re-express $\hat{a}_i$ in the new round β estimate to obtain another round estimate of β. Repeat these steps until the successive difference between estimates of Θ becomes sufficiently small. This process provides an efficient method to estimate Θ.

[11] Standard errors of $\hat{\Theta}$ presented here in column 3 are conditional on $(a_i : i = 1, \ldots, N)$ for which we substitute $(\hat{a}_i : i = 1, \ldots, N)$. This substitution affects the standard errors of $\hat{\Theta}$ if $\hat{\Theta}$ and $\hat{a}_i$ are correlated. However, their correlation is expected to be small, as explained in footnote 9. Thus we use the unmodified estimates of the standard errors of $\hat{\Theta}$.

[12] See Topel (1991) for reasons why tenure effects are potentially underestimated in the PSID panel data.

Table I. Estimates of incomplete worker information, incomplete firm information, and two-tier earnings functions parameters[a]

	OLS (initial points)	Heterogeneity unadjusted MLE[b]	Heterogeneity adjusted MLE[b]
Constant	−0·021741	0·01639	–[c]
	(−0·82)	(6·10)	–
EDUC	0·069096	0·06775	–[c]
	(51·81)	(79·61)	–
EXPER	0·036916	0·035714	0·036285
	(25·21)	(35·19)	(100·43)
$EXPER^2$	−0·000581	−0·000564	−0·000544
	(−20·39)	(−24·91)	(−60·06)
TENURE	0·000852	0·000733	0·000194
	(7·45)	(9·92)	(3·17)
$TENURE^2$	−0·17E − 5	−0·133E − 5	−0·543E − 6
	(−5·16)	(−6·53)	(3·17)
$\hat{\Theta}_u$		3·9538	31·036
		(30·89)	(10·82)
$\hat{\Theta}_v$		0·99094	0·16617
		(17·54)	(10·48)
$\hat{\Theta}_w$		1·0111	0·18123
		(19·75)	(10·61)
log L		−7733·35	−311·46
$\hat{\sigma}_u$		0·25292	0·0322
$\hat{\mu}_v$		0·25523	0·19390
$\hat{\mu}_w$		0·25014	0·17779

[a] PSID data for white males consisting of 13,408 observations for 838 individuals over 16 time periods, 1969–84.
[b] IMSL program used for likelihood function maximization.
[c] No coefficient is estimated since the variable does not change over the 16-year time period.
Asymptotic t-values in parentheses.

unexpected. Accounting for heterogeneity decreases the normal two-sided error dispersion as this two-sided error without the heterogeneity correction picks up most of the heterogeneity. The dispersion of v^*_{it} and w^*_{it}, μ_v and μ_w, decrease because they now represent market level worker and firm (supply and demand) inefficiency (incomplete information) devoid of individual heterogeneity. The time-invariant individual worker and firm information levels have been embedded in the person-specific constant.[13] The improved fit, as indicated by large reductions in the three errors' dispersion measures, testifies to the importance of unobserved heterogeneity correction in the frontier inefficiency model.

[13] Unlike for v^*_{it} and w^*_{it}, the estimation technique does not restrict v_i and w_i to be less than and greater than zero respectively, as hypothesized in equations (5) and (6). However, it is possible to surmise that these restrictions hold. Although contaminated by unobserved individual heterogeneity, column 2's (Table I) estimates of μ_v and μ_w contain *both* v^*_{it} and v_i and w_i and w^*_{it}. Assuming average unobserved heterogeneity to be zero for most attributes, then the remaining heterogeneity represents v_i and w_i. The v_i and w_i values are of the correct sign since netting these out (column 3) reduces worker and firm incomplete information. Depending on the extent of heterogeneity the differences in incomplete information values between columns 2 and 3 reflect average fixed effect incomplete information inefficiency.

5. CONCLUSIONS

This paper makes headway in distinguishing unobserved heterogeneity from inefficiency in a generalized two-tier frontier estimation model that contains the more traditional one-tier frontier as a special case. It does so by using a maximum likelihood panel data technique to obtain marketwide inefficiency measures in addition to heterogeneity-free coefficient estimates. The techniques are illustrated for 1968–84 PSID panel data on white males. Where possible, comparisons were made with coefficient estimates from past studies. These yielded surprising conformity. The traditional quadratic age–earnings profile was preserved, accounting for heterogeneity decreased the tenure–wage gradient, employees had slightly less information than employers, and the overall statistical fit increased markedly.

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REFERENCES

Aigner, D. J., C. A. K. Lovell, and P. Schmidt (1977), 'Frontier estimation of stochastic frontier production models', *Journal of Econometrics*, **6**,1 (July), 21–37.

Altonji, J. and R. Shakotko (1987), 'Do wages rise with job seniority?' *Review of Economic Studies*, **54**, 437–469.

Cornwell, C., P. Schmidt, and R. Sickles (1990), 'Production frontiers with cross-sectional and time-series variation in efficiency levels', *Journal of Econometrics*, **46**, 185–200.

Farrell, M. J. (1957), 'The measurement of productive efficiency', *Journal of the Royal Statistical Society*, Series A, General, **120**, Part 3, 253–290.

Gaynor, M. and S. Polachek (1994), 'Measuring information in the market: an application to physician services', *Southern Economic Journal*, **60**(4), 815–831.

Gong, B. and R. Sickles (1989), 'Finite sample evidence on the performance of stochastic frontier models using panel data', *The Journal of Productivity Analysis*, **1**, 229–261.

Heshmati, A. and S. Kumbhakar (1994), 'Farm heterogeneity and technical efficiency: a panel data model of Swedish dairy farms', *Journal of Productivity Analysis*, **5**, 45–61.

Kumbhakar, S. (1991), 'Estimation of technical inefficiency in panel data models with firm and time-specific effects', *Economics Letters*, **36**, 43–48.

Kumbhakar, S. and L. Hjalmarsson (forthcoming), 'Labour use efficiency in Swedish social insurance offices', *Journal of Applied Econometrics*.

Meeusen, W. and J. van den Broeck (1977), 'Efficiency estimation from Cobb–Douglas production functions with composed errors', *International Economic Review*, **18**, 2 (June), 435–444.

Pitt, M. and L. F. Lee (1981), 'The measurement and sources of technological inefficiency in the Indonesian weaving industry', *Journal of Development Economics*, **9**, **1** (August), 43–64.

Polachek, S. W. and M. Kim (1994), 'Panel estimates of the gender earnings gap: individual specific intercept and individual specific slope models', *Journal of Econometrics*, **61**, **1** (March), 23–42.

Polachek, S. W. and F. Horvath (1977), 'A life cycle approach to migration: analysis of the perspicacious peregrinator', *Research in Labour Economics*, **1**, 103–149.

Polachek, S. W. and B. J. Yoon (1985), 'Frontier estimation: a three-error component generalization', Working paper, SUNY-Binghamton.

Polachek, S. W. and B. J. Yoon (1987), 'A two-tiered earnings frontier estimation of employer and employee information in the labor market', *The Review of Economics and Statistics*, **69**, 2 (May), 296–302.

Polachek, S. W. and B. J. Yoon (1994), 'Estimating a two-tiered earnings frontier', Working paper, SUNY-Binghamton.

Ruhm, C. J. (1990), 'Do earnings increase with job seniority?' *The Review of Economics and Statistics*, **72**, 1 (February), 143–147.

Schmidt P. and R. Sickles (1984), 'Production frontiers with panel data', *Journal of Business and Economic Statistics*, **2**, 4, 367–374.

Topel, R. (1991), 'Specific capital mobility, and wages: wages rise with job seniority', *The Journal of Political Economy*, **99**, No. 1 (February) 145–176.

Yoon, B. J. and S. W. Polachek (1987), 'Employee and employer information in the labor market', in B. J. Ahn *et al.* (eds) *Theory and Practice of Economic Development: Essays in Honor of Professor Hyung Yoon Byun*, Bee Bong Publishing Company, Seoul, Korea.

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[1] **Measuring Information in the Market: An Application to Physician Services**
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[12] **Specific Capital, Mobility, and Wages: Wages Rise with Job Seniority**
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Stable URL:
http://links.jstor.org/sici?sici=0022-3808%28199102%2999%3A1%3C145%3ASCMAWW%3E2.0.CO%3B2-L

References

Do Wages Rise with Job Seniority?
Joseph G. Altonji; Robert A. Shakotko
The Review of Economic Studies, Vol. 54, No. 3. (Jul., 1987), pp. 437-459.
Stable URL:
http://links.jstor.org/sici?sici=0034-6527%28198707%2954%3A3%3C437%3ADWRWJS%3E2.0.CO%3B2-4

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Stable URL:
http://links.jstor.org/sici?sici=0020-6598%28197706%2918%3A2%3C435%3AEEFCPF%3E2.0.CO%3B2-K

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Christopher J. Ruhm
The Review of Economics and Statistics, Vol. 72, No. 1. (Feb., 1990), pp. 143-147.
Stable URL:
http://links.jstor.org/sici?sici=0034-6535%28199002%2972%3A1%3C143%3ADEIWJS%3E2.0.CO%3B2-T

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